Distance between H and Cl atoms = 1.27Å

Mass of H atom = m

Mass of Cl atom = 35.5m

Let the centre of mass of the system lie at a distance x from the Cl atom.

Distance of the centre of mass from the H atom = (1.27 – x)

Let us assume that the centre of mass of the given molecule lies at the origin. Therefore, we can have:

$$\frac{m(1.27 - x) + 35.5mx}{m + 35.5m} = 0$$

$$m(1.27 - x) + 35.5mx = 0$$
$$1.27 - x = -35.5x$$

$$\therefore x = \frac{-1.27}{(35.5 - 1)} = -0.037\,\text{Å}$$

Here, the negative sign indicates that the centre of mass lies at the left of the molecule. Hence, the centre of mass of the HCl molecule lies 0.037Å from the Cl atom.

Question 7.3:

A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?

ANS:

No change

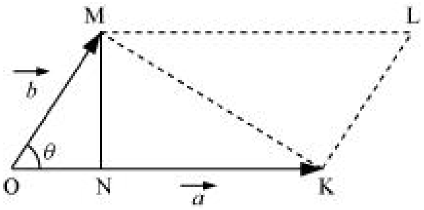
The child is running arbitrarily on a trolley moving with velocity v. However, the running of the child will produce no effect on the velocity of the centre of mass of the trolley. This is because the force due to the boy's motion is purely internal. Internal forces produce no effect on the motion of the bodies on which they act. Since no external force is involved in the boy–trolley system, the boy's motion will produce no change in the velocity of the centre of mass of the trolley.

Question 7.4:

Show that the area of the triangle contained between the vectors a and b is one half of the magnitude of a × b.

ANS:

Consider two vectors $\overline{\mathrm{OK}} = |\vec{a}|$ and $\overline{\mathrm{OM}} = |\vec{b}|$, inclined at an angle θ, as shown in the following figure.



In ΔOMN, we can write the relation:

$$\sin\theta = \frac{\mathrm{MN}}{\mathrm{OM}} = \frac{\mathrm{MN}}{|\vec{b}|}$$

$$\mathrm{MN} = |\vec{b}|\sin\theta$$

$$|\vec{a} \times \vec{a}| = |\vec{a}|\ |\vec{b}|\sin\theta$$

$$= \mathrm{OK} \cdot \mathrm{MN} \times \frac{2}{2}$$

= 2 × Area of ΔOMK

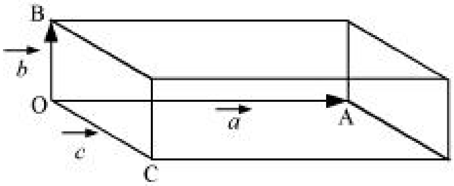
Area of ΔOMK $= \frac{1}{2}|\vec{a} \times \vec{b}|$

Question 7.5:

Show that a. (b × c) is equal in magnitude to the volume of the parallelepiped formed on the three vectors, a, b and c.

ANS:

A parallelepiped with origin O and sides a, b, and c is shown in the following figure.



Volume of the given parallelepiped = abc

$$\overrightarrow{OC} = \vec{a}$$
$$\overrightarrow{OB} = \vec{b}$$
$$\overrightarrow{OC} = \vec{c}$$

Let $\hat{\mathbf{n}}$ be a unit vector perpendicular to both b and c. Hence, $\hat{\mathbf{n}}$ and a have the same direction.

$$\vec{b} \times \vec{c} = bc\sin\theta\ \hat{\mathbf{n}}$$

$$= bc\sin 90°\ \hat{\mathbf{n}}$$
$$= bc\,\hat{n}$$

$$\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$$
$$= a\cdot\left(bc\,\hat{\mathbf{n}}\right)$$

= abc cosθ $\hat{\mathbf{n}}$

= abc cos 0°

= abc

= Volume of the parallelepiped

Question 7.6:

Find the components along the x, y, z axes of the angular momentum l of a particle, whose position vector is r with components x, y, z and momentum is p with components p_x, p_y and p_z. Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

ANS:

$l_x = yp_z - zp_y$

$l_y = zp_x - xp_z$

$l_z = xp_y - yp_x$

Linear momentum of the particle, $\vec{p} = p_x \hat{\mathbf{i}} + p_y \hat{\mathbf{j}} + p_z \hat{\mathbf{k}}$

Position vector of the particle, $\vec{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

Angular momentum, $\vec{l} = \vec{r} \times \vec{p}$

$$= \left(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}\right) \times \left(p_x \hat{\mathbf{i}} + p_y \hat{\mathbf{j}} + p_z \hat{\mathbf{k}}\right)$$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$l_x \hat{\mathbf{i}} + l_y \hat{\mathbf{j}} + l_z \hat{\mathbf{k}} = \hat{\mathbf{i}}\left(yp_z - zp_y\right) - \hat{\mathbf{j}}\left(xp_z - zp_x\right) + \hat{\mathbf{k}}\left(xp_y - zp_x\right)$$

Comparing the coefficients of $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \text{and } \hat{\mathbf{k}},$ we get:

$$\left.\begin{aligned} l_x &= yp_z - zp_y \\ l_y &= xp_z - zp_x \\ l_z &= xp_y - yp_x \end{aligned}\right\} \qquad \ldots (i)$$

The particle moves in the x-y plane. Hence, the z-component of the position vector and linear momentum vector becomes zero, i.e.,

$z = p_z = 0$

Thus, equation (i) reduces to:

$$\left.\begin{aligned} l_x &= 0 \\ l_y &= 0 \\ l_z &= xp_y - yp_x \end{aligned}\right\}$$

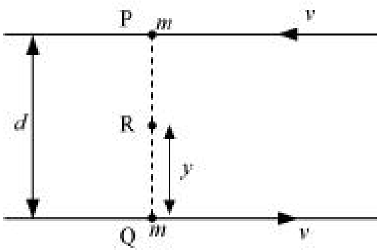
Therefore, when the particle is confined to move in the x-y plane, the direction of angular momentum is along the z-direction.

Question 7.7:

Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken.

ANS:

Let at a certain instant two particles be at points P and Q, as shown in the following figure.



Angular momentum of the system about point P:

$$\begin{aligned} \vec{L}_P &= mv \times 0 + mv \times d \\ &= mvd \qquad \ldots (i) \end{aligned}$$

Angular momentum of the system about point Q:

$$\begin{aligned} \vec{L}_Q &= mv \times d + mv \times 0 \\ &= mvd \qquad \ldots (ii) \end{aligned}$$

Consider a point R, which is at a distance y from point Q, i.e.,

QR = y

∴PR = d – y

Angular momentum of the system about point R:

$$\vec{L}_R = mv \times (d - y) + mv \times y$$
$$= mvd - mvy + mvy$$
$$= mvd \qquad \text{... } (iii)$$

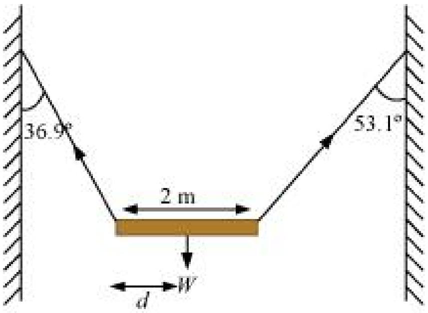
Comparing equations (i), (ii), and (iii), we get:

$$\vec{L}_P = \vec{L}_Q = \vec{L}_R \qquad \text{... } (iv)$$

We infer from equation (iv) that the angular momentum of a system does not depend on the point about which it is taken.

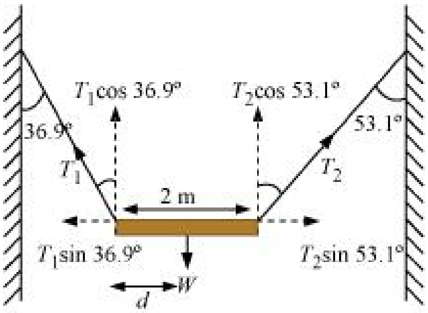
Question 7.8:

A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig.7.39. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.



ANS:

The free body diagram of the bar is shown in the following figure.



Length of the bar, l = 2 m

T_1 and T_2 are the tensions produced in the left and right strings respectively.

At translational equilibrium, we have:

$$T_1 \sin 36.9° = T_2 \sin 53.1$$

$$\frac{T_1}{T_2} = \frac{\sin 53.1°}{\sin 36.9}$$

$$= \frac{0.800}{0.600} = \frac{4}{3}$$

$$\Rightarrow T_1 = \frac{4}{3} T_2$$

For rotational equilibrium, on taking the torque about the centre of gravity, we have:

$$T_1 \cos 36.9 \times d = T_2 \cos 53.1(2-d)$$

$$T_1 \times 0.800 d = T_2\ 0.600(2-d)$$

$$\frac{4}{3} \times T_2 \times 0.800 d = T_2\left[0.600 \times 2 - 0.600 d\right]$$

$$1.067 d + 0.6 d = 1.2$$

$$\therefore d = \frac{1.2}{1.67}$$

$$= 0.72 \text{ m}$$

Hence, the C.G. (centre of gravity) of the given bar lies 0.72 m from its left end.

Question 7.9:

A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

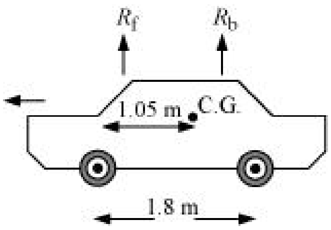
ANS:

Mass of the car, m = 1800 kg

Distance between the front and back axles, d = 1.8 m

Distance between the C.G. (centre of gravity) and the back axle = 1.05 m

The various forces acting on the car are shown in the following figure.



R_f and R_b are the forces exerted by the level ground on the front and back wheels respectively.

At translational equilibrium:

$R_f + R_b$ = mg

= 1800 × 9.8

= 17640 N … (i)

For rotational equilibrium, on taking the torque about the C.G., we have:

$$R_f(1.05) = R_b(1.8 - 1.05)$$
$$R_f \times 1.05 = R_b \times 0.75$$

$$\frac{R_f}{R_b} = \frac{0.75}{1.05} = \frac{5}{7}$$

$$\frac{R_b}{R_f} = \frac{7}{5}$$

$$R_b = 1.4\ R_f \qquad \dots(ii)$$

Solving equations (i) and (ii), we get:

$$1.4R_f + R_f = 17640$$

$$R_f = \frac{17640}{2.4} = 7350\ \text{N}$$

$\therefore R_b$ = 17640 – 7350 = 10290 N

Therefore, the force exerted on each front wheel $= \frac{7350}{2} = 3675\ \text{N}$, and

The force exerted on each back wheel $= \frac{10290}{2} = 5145\ \text{N}$

Question 7.10:

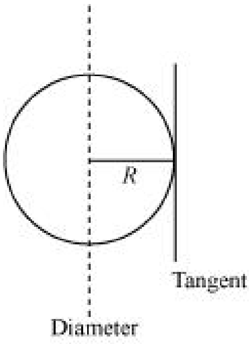
(a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.

(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

ANS:

(a) $\frac{7}{5}MR^2$

The moment of inertia (M.I.) of a sphere about its diameter $= \frac{2}{5}MR^2$



M.I. $= \frac{2}{5}MR^2$

According to the theorem of parallel axes, the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

The M.I. about a tangent of the sphere $= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

(b) $\frac{3}{2}MR^2$

The moment of inertia of a disc about its diameter = $\frac{1}{4}MR^2$